

Errata for *Real Analysis for Graduate Students*,
Version 2.1

NOTICE: A later version of *Real Analysis for Graduate Students* is now available for free download: go to

<http://www.math.uconn.edu/~bass/real.html>

The errata pages for Version 2.1 will no longer be updated effective January 6, 2016.

Page 15, line -4: Add at end:
Exercise 3.8 guarantees that the completion of \mathcal{A} exists.

Page 20, line -7: $\emptyset \in \mathcal{C}$

Page 23, line -8: $A_i = C_i \cap (C_1 \cup \cdots \cup C_{i-1})^c$

Page 25, line 13: $a_{k_1} < C < b_{k_1}$.

Page 27, line 11: ℓ instead of ℓ^*

Page 28, line -5: jump discontinuity

Page 29, line -14: be a Lebesgue measurable set

Page 29, line -9: is a Lebesgue measurable set.

Page 39, line -14: $\inf_j \sup_{i \geq j} f_i$

Page 39, line -13: $= \bigcup_{i=1}^{\infty} \{x : f_i(x) > a\}$,

Page 42, line 8: is Lebesgue measurable

Page 42, line 14: is a Lebesgue

Page 42, line -1:

$$\sum_{i=1}^{K2^n+1} \frac{i-1}{2^n} \chi_{A_{i_n}}(x)$$

Page 48, line -8:

(3) If f is real and integrable and c is real, then $\int cf \, d\mu = c \int f \, d\mu$.

Page 49, lines 3–14: move to just above Page 54, line -2.

Page 51, lines -5,-4: increasing sequence. Let L

Page 52, line 3: $c \int_{A_n} s$

Page 54, line 8: Add:

Proposition 7.4A. *If f is integrable and c is complex, then $\int cf = c \int f$.*

Proof. Combine Proposition 6.3(3) and Theorem 7.4. \square

Page 55, line -5: $= \int f d\mu$

Page 57, line 1: f and each f_n is

Page 58, Exercise 7.15: of measurable real-valued

Page 60, Exercise 7.23: finite positive measures

Page 61, line 2: $\alpha_{n_j}(r)$

Page 61, line 4: $\bar{\alpha}(r) = \lim_{j \rightarrow \infty} \alpha_{n_j}(r)$

Page 61, line -2: $\int f d\mu_{n_j}$

Page 63, line -9: real-valued and integrable

Page 66, line 19: $= |a|^{-1} \int_{\mathbb{R}} f(x) dx.$

Page 84, line 16: $\nu_i(B) = \nu(B \cap G_i)$

Page 84, line 18: Add after “ $k_i(y)$.”:

Check that

$$\int h_i(x) \mu_i(dx) = \int h_i(x) \chi_{F_i}(x) \mu(dx)$$

and similarly with h_i and μ_i replaced by k_i and ν_i .

Page 88, lines 5 and 6: $\int \int f(x, y) \mu(dx) \mu(dy)$

Page 94, line -6: measurable subsets of E with

Page 95, line 5: sum converges. This implies $\mu(E_k) \rightarrow 0$, and so $n_k \rightarrow \infty$.

Page 95, line -7: (1) If there is no negative set, there is nothing to prove.
Let L

Page 95, line -6: Let $B_1 = A_1$ and let

Page 96, line 3: Add at end: Hence $L > -\infty$.

- Page 101, line -2: note $L \leq \nu(X) < \infty$, and let g_n
- Page 105, Exercise 13.8: $\rho \ll \nu$. Here $\nu \ll \mu$ means that $\nu(A) = 0$ whenever $\mu(A) = 0$ and A is in the σ -algebra. Prove that $\rho \ll \mu$
- Page 109, line 10: $B_\alpha \subset B_{j_0}^*$
- Page 109, line 16: of f . Observe that in the definition we could equally well take the supremum over positive rationals without changing the value of $Mf(x)$, hence Mf is the supremum of a countable number of continuous functions, and thus is measurable.
- Page 109, line -5: $\int_{B_x} |f(y)| dy$
- Page 110, line -9: Using Theorem 8.4 (or rather, the obvious multidimensional analogue of Theorem 8.4),
- Page 112, line -9: $\frac{1}{m(B(x,r))}$
- Page 119, line -8: Lemma 14.10, so that f_1 and f_2
- Page 131, line -8: $= \inf\{M \geq 0 :$
- Page 131, line -7: If no such M exists, then $\|f\|_\infty = \infty$. Thus the
- Page 135, line 13: $\leq \left(\frac{1}{2} + \|f_{n_1}\|_p\right)^p$
- Page 135, line -1: $\leq 2^{-(j+1)p}$
- Page 136, line -4: with respect to the
- Page 136, line -2: In this section only, all functions are defined on \mathbb{R}^n and we are using Lebesgue measure on \mathbb{R}^n .
- Page 138, line 3: Add at the end: suppose $f \in L^p$. Then
- Page 139, line 9: Add at end: suppose f is measurable, and $\int fg$ is defined for all simple g . Then
- Page 140, line 2: Note $\chi_A \in L^p$. We will
- Page 140, line 3: ν is a signed measure.
- Page 140, line 15: $= \int_A g d\mu$
- Page 160, line -4: $\ell(G_i) \leq \mu^*(A_i) + \varepsilon 2^{-i}$

- Page 186, line -7: $\lambda x + (1 - \lambda)y \in E$
- Page 192, line 11: of B . Suppose $V \neq H$. Choose non-zero $x \in V^\perp$,
- Page 192, line 17: larger orthonormal set.
- Page 193, line 16: $\tilde{\mathcal{F}}$
- Page 193, line 19: If $e^{i\theta_1}, e^{i\theta_2} \in S$ and $e^{i\theta_1} \neq e^{i\theta_2}$, then
- Page 193, line 21:
$$\frac{\tilde{u}_1(e^{i\theta_1})}{\tilde{u}_1(e^{i\theta_2})} =$$
- Page 193, line 22: $\tilde{u}_1(e^{i\theta_1}) \neq \tilde{u}_1(e^{i\theta_2})$
- Page 194, line 5: Replace \bar{f} by f in all three places.
- Page 228, line 8: same G works
- Page 297, Exercise 21.24, line 6: $\max_{1 \leq k \leq n} S_k$
- Page 298, Exercise 21.28: Change $2n$ to 2^n twice
- Page 298, Exercise 21.31: $d(\mathbb{P}, \mathbb{Q}) = \sup\{$
- Page 299, Exercise 21.37: not necessarily identically distributed,

Special thanks to Richard Laugesen for pointing out many of the corrections above.