Errata for *Real Analysis for Graduate Students*,
*Version 3.1*

Page 16, Exercise 2.9, line 2: \( \lim \inf_i A_i = \)

Page 16, Exercise 2.9(3): Given a set

Page 41, Exercise 4.18: numbers

Page 50, Exercise 5.6(2): Borel measurable

Page 50, Exercise 5.6(2): from \( \mathbb{C} \) to the unit circle.

Page 54, lines 17–24: Replace by the following.

**Proposition 6.3**  (1) If \( f \) is a real-valued measurable function with \( 0 \leq a \leq f(x) \leq b \) for all \( x \) and \( \mu(X) < \infty \), then \( a \mu(X) \leq \int f \, d\mu \leq b \mu(X) \);

(2) If \( f \) and \( g \) are measurable, real-valued, and integrable and \( 0 \leq f(x) \leq g(x) \) for all \( x \), then \( \int f \, d\mu \leq \int g \, d\mu \).

(3) If \( f \) is real-valued, non-negative, and integrable and \( c \) is a non-negative real number, then \( \int cf \, d\mu = c \int f \, d\mu \).

(4) If \( \mu(A) = 0 \) and \( f \) is non-negative and measurable, then \( \int f \chi_A \, d\mu = 0 \).

Page 60, lines 7-10: Replace by the following.

**Proposition 7.5**  (1) If \( f \) is a real-valued measurable function with \( a \leq f(x) \leq b \) for all \( x \) and \( \mu(X) < \infty \), then \( a \mu(X) \leq \int f \, d\mu \leq b \mu(X) \);

(2) If \( f \) and \( g \) are measurable, real-valued, and integrable and \( f(x) \leq g(x) \) for all \( x \), then \( \int f \, d\mu \leq \int g \, d\mu \).

(3) If \( f \) is complex-valued and integrable and \( c \) is a complex number, then \( \int cf \, d\mu = c \int f \, d\mu \).

(4) If \( \mu(A) = 0 \) and \( f \) is measurable, then \( \int f \chi_A \, d\mu = 0 \).

**Proof.** These follow from the definition of the Lebesgue integral of a complex-valued function, Proposition 6.3, and Theorem 7.4. For example, to prove (2), write \( f = f^+ - f^- \) and \( g = g^+ - g^- \). Then

\[
 f^+(x) - f^-(x) = f(x) \leq g(x) = g^+(x) - g^-(x)
\]

implies

\[
 0 \leq f^+(x) + g^-(x) \leq g^+(x) + f^-(x)
\]

for all \( x \). Proposition 6.3(2) implies \( \int (f^+ + g^-) \leq \int (g^+ + f^-) \), and the linearity of the Lebesgue integral implies that \( \int f \leq \int g \). \( \square \)
Page 63, line 1: if $f$ and the $f_n$ are

Page 63, Exercise 7.4, line 2: its integral is equal

Page 64, Exercise 7.14:

$$\lim_{n \to \infty} \int_0^\infty ne^{-nx} \frac{x^2 + 1}{x^2 + x + 1} \, dx.$$  

Page 71, line 4: $f$ is a Lebesgue measurable

Page 71, line 13: to $\int f$, which is finite, so taking

Page 71, line 17: $A$ is a bounded Lebesgue measurable

Page 73, line -2: (3) Continue to suppose $\mu(X) = 1$. Deduce

Page 81, line 3: some of them. All functions in this chapter are assumed to be measurable.

Page 91, line -4: $\nu(dy)$

Page 95, Exercise 11.7, line 4: Only one “the”

Page 97, Exercise 11.17, line 2: Suppose $\mu$ and $\nu$ are $\sigma$-finite. Prove

Page 99, Exercise 11.23: $(x, y) \in A$

Page 103, line -4: (1) Note that there is at least one negative set, namely, $\emptyset$.

Page 119, lines 5–6: Replace “Therefore . . . for some $j_0 \leq k$.” by “Let $j_0$ be the smallest positive integer less than or equal to $k$ such that $B_\alpha$ intersects $B_{j_0}$."

Page 135, line 9: $A = E_{uv} \cap (\cup_{n=1}^N (x_n - h_n, x_n))$

Page 138, line 18: Change “This implies” to “Since almost every point of $[a, c]$ is in $E$, this implies”

Page 139, Exercise 14.1(2): $f(b)g(b)$

Page 144, line 5: extraneous apostrophe

Page 145, line 4: The cases $p = 1$ and $a = 0$ are obvious, so we assume $p > 1$ and $a > 0$.  

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Page 146, line 21: Extraneous period at end.

Page 161, Exercise 15.30: Define $\varphi(0) = 0$ and $\varphi(x) = c_1 e^{-1/|x|^2}$ for $x \neq 0$.

Page 185, Exercise 17.11: $\sup_{f \in C(X), \sup |f| \leq 1}$

Page 189, line 13: where $x \in M$

Page 196, line 2: Replace $f$ by $x$ twice, $g$ by $y$ twice

Page 248, line 10: analysis class