Errata for *Real Analysis for Graduate Students,*

Version 3.1

Page 16, Exercise 2.9, line 2: $\liminf_i A_i =$

Page 16, Exercise 2.9(3): Given a set

Page 41, Exercise 4.18: numbers

Page 50, Exercise 5.6(2): Borel measurable

Page 50, Exercise 5.6(2): from $\mathbb{C}$ to the unit circle.

Page 54, lines 17–24: Replace by the following.

Proposition 6.3 (1) If f is a real-valued measurable function with $0 \le a \le f(x) \le b$ for all x and $\mu(X) < \infty$, then $a\mu(X) \le \int f\, d\mu \le b\mu(X)$;

(2) If f and g are measurable, real-valued, and integrable and $0 \le f(x) \le g(x)$ for all x, then $\int f\, d\mu \le \int g\, d\mu$.

(3) If f is real-valued, non-negative, and integrable and c is a non-negative real number, then $\int cf\, d\mu = c \int f\, d\mu$.

(4) If $\mu(A) = 0$ and f is non-negative and measurable, then $\int f\chi_A\, d\mu = 0$.

Page 60, lines 7-10: Replace by the following.

Proposition 7.5 (1) If f is a real-valued measurable function with $a \le f(x) \le b$ for all x and $\mu(X) < \infty$, then $a\mu(X) \le \int f\, d\mu \le b\mu(X)$;

(2) If f and g are measurable, real-valued, and integrable and $f(x) \le g(x)$ for all x, then $\int f\, d\mu \le \int g\, d\mu$.

(3) If f is complex-valued and integrable and c is a complex number, then $\int cf\, d\mu = c \int f\, d\mu$.

(4) If $\mu(A) = 0$ and f is measurable, then $\int f\chi_A\, d\mu = 0$.

Proof. These follow from the definition of the Lebesgue integral of a complex-valued function, Proposition 6.3, and Theorem 7.4. For example, to prove (2), write $f = f^+ - f^-$ and $g = g^+ - g^-$. Then

$$f^+(x) - f^-(x) = f(x) \le g(x) = g^+(x) - g^-(x)$$

implies

$$0 \le f^+(x) + g^-(x) \le g^+(x) + f^-(x)$$

for all x. Proposition 6.3(2) implies $\int (f^+ + g^-) \le \int (g^+ + f^-)$, and the linearity of the Lebesgue integral imples that $\int f \le \int g$. $\square$

Page 63, line 1: if f and the f_n are

Page 63, Exercise 7.4, line 2: its integral is equal

Page 64, Exercise 7.14:

$$\lim_{n\to\infty}\int_0^\infty ne^{-nx}\frac{x^2+1}{x^2+x+1}\,dx.$$

Page 71, line 4: f is a Lebesgue measurable

Page 71, line 13: to $\int f$, which is finite, so taking

Page 71, line 17: A is a bounded Lebesgue measurable

Page 73, line -2: (3) Continue to suppose $\mu(X)=1$. Deduce

Page 81, line 3: some of them. All functions in this chapter are assumed to be measurable.

Page 91, line -4: $\nu(dy)$

Page 95, Exercise 11.7, line 4: Only one "the"

Page 97, Exercise 11.17, line 2: Suppose μ and ν are σ-finite. Prove

Page 99, Exercise 11.23: $(x,y)\in A$

Page 103, line -4: (1) Note that there is at least one negative set, namely, $\emptyset$.

Page 119, lines 5–6: Replace "Therefore …for some $j_0\le k$." by "Let j_0 be the smallest positive integer less than or equal to k such that B_α intersects B_{j_0}."

Page 135, line 9: $A=E_{uv}\cap(\cup_{n=1}^N(x_n-h_n,x_n))$

Page 138, line 18: Change "This implies" to "Since almost every point of $[a,c]$ is in E, this implies"

Page 139, Exercise 14.1(2): $f(b)g(b)$

Page 144, line 5: extraneous apostrophe

Page 145, line 4: The cases $p=1$ and $a=0$ are obvious, so we assume $p>1$ and $a>0$.

Page 146, line 21: Extraneous period at end.

Page 161, Exercise 15.30: Define $\varphi(0) = 0$ and $\varphi(x) = c_1 e^{-1/|x|^2}$ for $x \neq 0$,

Page 185, Exercise 17.11: $\sup_{f \in \mathcal{C}(X),\, \sup|f| \leq 1}$

Page 189, line 13: where $x \in M$

Page 196, line 2: Replace f by x twice, g by y twice

Page 248, line 10: analysis class