Errata for *Stochastic Processes*

Page 4, line 7: Borel subset of $\mathcal{U}^n$ such that

Page 7, line -9: Delete "and if $s < t$,"

Page 8, line 4: independent of $Z_1, \ldots, Z_{i-1}$.

Page 8, line 22: $= \mathrm{Var}\, W_t - 2\mathrm{Cov}\,(W_s, W_t) + \mathrm{Var}\, W_s$

Page 12, Problem 2.7: $X = Z$ a.s.

Page 13, line -9: Change first "is" to "if"

Page 14, line 19: Then if $s < t$,

Page 14, line -5: $\mathbb{E}\,[\sup_{s \le t} |M_s|^p]$

Page 14, line -3: $\mathcal{D}_n = \{kt/2^n : 0 \le k \le 2^n, k \in \mathbb{Z}\}$

Page 15, line 19: equal to $X(T(\omega), \omega)$ when $T(\omega) < \infty$;

Page 16, line 4: $(T_A < t) = \cup_{q \in \mathbb{Q}, q < t}(X_q \in A)$

Page 16, lines 8-11: Replace "Since $A \subset A_n$ ..." to the end of the proof by the following:

Let $U_A = \inf\{t \ge 0 : X_t \in A\}$. (Note here we allow $t \ge 0$ compared to $t > 0$ in the definition of T_A.) Since $A \subset A_n$ and A_n is open, then $U_A \ge T_{A_n}$, so $U_A \ge T$. Because X has continuous paths, on the event $(T < \infty)$ we have $X_T = \lim_n X_{T_{A_n}}$. If $n \ge m$, then $X(T_{A_n}) \in \overline{A_n} \subset \overline{A_m}$. Therefore $X_T \in \overline{A_m}$ for each m. Snce $A = \cap_m \overline{A_m}$, then $X_T \in A$. Therefore $U_A \le T$ on the event $(T < \infty)$. Since $U_A \ge T$, we have $U_A = T$ on the event $(T = \infty)$, so in any case $U_A = T$, a stopping time.

Apply the above paragraph to $X^n_t = X_{t+\frac{1}{n}}$, call the corresponding stopping times U^n_A, and let $V^n_A = U^n_A + \frac{1}{n}$. By Proposition 3.8 V^n_A is a stopping time. Observe that V^n_A is the first time greater than or equal to $1/n$ (possibly including the time $1/n$) that $X_t \in A$, and it follows that $V^n_A \downarrow T_A$. Therefore T_A is a stopping time. □

Page 16, line 19: each $t \geq 0$

Page 17, line 13: Theorem 3.6(2)

Page 17, line -9: $\mathcal{D}_n = \{k/2^n : k \geq 0, k \in \mathbb{Z}\}$

Page 18, line 1: (Theorem A.32)

Page 33, line 4: $2^n \mathbb{P}(X_{t_0/2^n} \geq 2)$

Page 33, line 5: "=" instead of "≤"

Page 55, line -4: Theorem 3.11

Page 56, line 3: By Exercise 3.12

Page 56, line 18: Then if $S < T$ are finite stopping times

Page 58, line 16: $\langle M \rangle_{t_0}$

Page 62, Exercise 9.4: $M_t^i = W_{t \wedge t_0}^i$

Page 64, line -5: missing right parenthesis after ω

Page 66, line 1: by (9.3)

Page 66, line 6: right parenthesis missing at end of line

Page 66, line -9: $K_j(\omega) 1_{(a_j, b_j]}(s)$

Page 67, lines 6–13: Replace each "H_j" by "K_j" and "H_i" by "K_i"

Page 79, (12.7): $-\langle M \rangle_{t_0}$

Page 80, line 17: $d(iuW_r + u^2 r/2)$

Page 82, line 9: $M_{t_0} = \int_0^{t_0} H_s \, dW_s$

Page 90, line 9: $C_r = \int_0^r H'_u 1_B \, dA'_u$

Page 91, line -9: $= \mathbb{E}_{\mathbb{P}}\Big[\int_s^t \mathbb{E}[M_t \mid \mathcal{F}_r] H_r \, dr; B\Big]$

Page 115, lines -17 to -7: Replace with the following:

(2) Now suppose A is compact and let $A_n = \{x \in \mathcal{S} : d(x, A) < 1/n\}$. Each set A_n is open, hence T_{A_n} is a stopping time for each n. The T_{A_n} increase; let T be the limit. If we show $T = U_A$, a.s., this will prove U_A is a stopping time.

Since $A \subset A_n$ and A_n is open, then $T_{A_n} \le U_A$ for each n. Therefore $T \le U_A$. On the other hand, on the event $(T < \infty)$, if $n > m$, then $X_{T_{A_n}} \in \overline{A}_n \subset \overline{A}_m$, the closure of A_m. Either $T_{A_n}(\omega) = T(\omega)$ for all n sufficiently large, in which case $X_T(\omega) \in \overline{A}_m$, or else $T_{A_n}(\omega) < T(\omega)$ for all n. In the latter case, $X_T(\omega) = \lim_{n\to\infty} X_{T_{A_n}}(\omega) \in \overline{A}_m$ except for ω's in a null set since the jump times of X are totally inaccessible. In either case, $X_T \in \overline{A}_m$. This is true for all m, so $X_T \in \cap_m \overline{A}_m = A$, again provided $T < \infty$, and therefore $U_A \le T$ on $(T < \infty)$. Since $U_A \ge T$, then we have $U_A = T$ both when $T(\omega) < \infty$ and when $T(\omega) = \infty$.

If we let $X_t^\delta = X_{t+\delta}$ and $U_A^\delta = \inf\{t \ge 0 : X_t^\delta \in A\}$, then by the above, U_A^δ is a stopping time with respect to the filtration $\{\mathcal{F}_t^\delta\}$, where $\mathcal{F}_t^\delta = \mathcal{F}_{t+\delta}$. It follows that $\delta + U_A^\delta$ is a stopping time with respect to the filtration $\{\mathcal{F}_t\}$. Since $(1/m) + U_A^{1/m} \downarrow T_A$, then T_A is a stopping time with respect to $\{\mathcal{F}_t\}$. $\square$

Page 140, line -5: V^n is integrable. Moreover $U_t^n = U_{T_n}^n$ and $V_t^n = V_{T_n}^n$ for $t \ge T_n$.

Page 140, line -4: U^n and V^n

Page 140, line -3: T_n on

Page 160, line -10: $(\mathbb{P}^x)^*(A)$

Page 180, line 15: $= A_{t-s} \circ \theta_s$.

Page 229, line -10: Let W_t be a one-dimensional Brownian motion, let h_t be a

Page 232, line 4: $d\langle M, \widetilde{Y} \rangle_t =$

Page 244, line -24: law of X under

Page 254, line -11: $c'\varepsilon^{-2/H}$

Page 254, line -10: again $\log N(\varepsilon) \le c \log(1/\varepsilon)$,

Page 261, line 1: $\rho(f, g) < \delta^2$,

Page 264, line -15: *in probability as $n \to \infty$. Then the X_n are tight.*

Page 279, line 5: $\mathcal{S}$. We make $\mathcal{S}$ compact by taking the one-point compactification. Let C_0

Page 288, line 1: one value of $\lambda > 0$, say

Page 288, line -5: $= \lambda R_\lambda f$.

Page 301, line 6: generator, and moreover $|\psi(u)| \le c(1 + |u|^2)$ for some constant c, then the Fourier

Page 334, line 2: $e^{-\lambda t}$ twice

Page 340, line -1: $\mathbb{P}(\sup_{s \le t} |X_s| \ge M) \le 1/2$.

Page 341, lines 3–7: Replace with the following.

$$\begin{aligned}\mathbb{P}(T_{i+1} < t) &\le \mathbb{P}(T_i < t, T_{i+1} - T_i < t) \\ &= \mathbb{P}(T_{i+1} - T_i < t)\mathbb{P}(T_i < t) \\ &= \mathbb{P}(T_1 < t)\mathbb{P}(T_i < t),\end{aligned}$$

using Lemma 42.2. Now

$$\mathbb{P}(T_1 < t) \le \mathbb{P}(\sup_{s \le t} |X_s| \ge M) \le \tfrac{1}{2},$$

so $\mathbb{P}(T_{i+1} < t) \le \frac{1}{2}\mathbb{P}(T_i < t)$, and then by induction, $\mathbb{P}(T_i < t) \le 2^{-i}$. Therefore

$$\mathbb{P}(\sup_{s \le t} |X_s| \ge 2(i+1)M) \le \mathbb{P}(T_i < t) \le 2^{-i}$$

and the

Page 374, line 5: $\frac{1}{2\sqrt{2\pi}} \cdot \frac{1}{x} e^{-x^2/2} \le P(Z \ge x)$

Page 382, line -2: by $B_n \times \mathcal{S}^{n-j_n}$

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